

Stats 2 Chi-Squared Contingency Table Tests Questions

- 2 Year 12 students at Newstatus School choose to participate in one of four sports during the Spring term.

The students' choices are summarised in the table.

	Squash	Badminton	Archery	Hockey	Total
Male	5	16	30	19	70
Female	4	20	33	53	110
Total	9	36	63	72	180

- (a) Use a χ^2 test, at the 5% level of significance, to determine whether the choice of sport is independent of gender. (10 marks)
- (b) Interpret your result in part (a) as it relates to students choosing hockey. (2 marks)
-

- 4 It is claimed that the area within which a school is situated affects the age profile of the staff employed at that school. In order to investigate this claim, the age profiles of staff employed at two schools with similar academic achievements are compared.

Academia High School, situated in a rural community, employs 120 staff whilst Best Manor Grammar School, situated in an inner-city community, employs 80 staff.

The **percentage** of staff within each age group, for each school, is given in the table.

Age	Academia High School	Best Manor Grammar School
22–34	17.5	40.0
35–39	60.0	45.0
40–59	22.5	15.0

- (a) (i) Form the data into a contingency table suitable for analysis using a χ^2 distribution. (2 marks)
- (ii) Use a χ^2 test, at the 1% level of significance, to determine whether there is an association between the age profile of the staff employed and the area within which the school is situated. (9 marks)
- (b) Interpret your result in part (a)(ii) as it relates to the 22–34 age group. (2 marks)
-

- 7 A statistics unit is required to determine whether or not there is an association between students' performances in mathematics at Key Stage 3 and at GCE.

A survey of the results of 500 students showed the following information:

		GCE Grade				Total
		A	B	C	Below C	
Key Stage 3 Level	8	60	55	47	43	205
	7	55	32	31	26	144
	6	40	38	35	38	151
Total		155	125	113	107	500

- (a) Use a χ^2 test at the 10% level of significance to determine whether there is an association between students' performances in mathematics at Key Stage 3 and at GCE. *(9 marks)*
- (b) Comment on the number of students who gained a grade A at GCE having gained a level 7 at Key Stage 3. *(1 mark)*
-
- 1 Two groups of patients, suffering from the same medical condition, took part in a clinical trial of a new drug. One of the groups was given the drug whilst the other group was given a placebo, a drug that has no physical effect on their medical condition.

The table shows the number of patients in each group and whether or not their condition improved.

	Placebo	Drug
Condition improved	20	46
Condition did not improve	55	29

Conduct a χ^2 test, at the 5% level of significance, to determine whether the condition of the patients at the conclusion of the trial is associated with the treatment that they were given. *(10 marks)*

Stats 2 Chi-Squared Contingency Table Tests Answers

2(a)	H_0 : Choice independent of gender	B1		gender not associated with choice															
	<table style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th></th> <th>Squash</th> <th>Badminton</th> <th>Archery</th> <th>Hockey</th> </tr> </thead> <tbody> <tr> <td>Male</td> <td>5/3.5</td> <td>16/14</td> <td>30/24.5</td> <td>19/28</td> </tr> <tr> <td>Female</td> <td>4/5.5</td> <td>20/22</td> <td>33/38.5</td> <td>53/44</td> </tr> </tbody> </table>		Squash	Badminton	Archery	Hockey	Male	5/3.5	16/14	30/24.5	19/28	Female	4/5.5	20/22	33/38.5	53/44	M1		
	Squash	Badminton	Archery	Hockey															
Male	5/3.5	16/14	30/24.5	19/28															
Female	4/5.5	20/22	33/38.5	53/44															
	Combine Squash and Badminton	M1		$E_i < 5$ (Similar categories)															
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	S & B	Archery	Hockey																
Male	21/17.5	30/24.5	19/28																
Female	24/27.5	33/38.5	53/44																
	χ^2 values																		
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Male	0.7000	1.2347	2.8928																
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	$\chi^2_{\text{calc}} = 7.90$	A1		(7.8 to 7.9)															
	$\nu = 2$	B1																	
	$\chi^2_{5\%}(2) = 5.991$	B1ft		(on their ν)															
	Reject H_0 Sufficient evidence, at the 1% level of significance, to support an association between the choice of sport and gender	A1ft	10	reject H_0 and H_0 stated or statement in context															
(b)	More females and fewer males chose to participate in hockey than expected	B1 B1	2																
	Total		12																

4(a)(i)

	A	B	Total
22-34	21	32	53
35-39	72	36	108
40-59	27	12	39
Total	120	80	200

B1
B1

2

for A values
for B values

(ii)

H_0 : no association between area
and age profile
 H_1 : association between area
and age profile

B1

At least H_0

O_i	E_i	$\frac{(O_i - E_i)^2}{E_i}$
24	31.8	3.6679
72	64.8	0.8000
24	23.4	0.5538
32	21.2	5.5019
36	43.2	1.2000
12	15.6	0.8308
$\sum O_i = 200$	$\sum E_i = 200$	$\chi^2 = 12.554$

M1

Attempt at Row & Column totals

M1

Attempt at E_i

M1

Attempt at $\frac{(O_i - E_i)^2}{E_i}$

M1

Attempt at χ^2

A1

AWFW 12.5 to 12.6 provided correct
method used

B1

$$\nu = (3-1)(2-1) = 2$$

B1✓

fit on their ν and χ^2

$$\chi_{1\%}^2(2) = 9.210 < 12.554$$

Reject H_0

The evidence suggests that the area within
which a school is situated seems to have an
effect on the age-profile of the staff
employed.

E1✓

9

fit on χ^2 and calculated value
depends on H_0 correct, if stated

(b) There seems to be fewer staff employed in
22 - 34 age group than expected in
school A
and more than expected in school B

E1

E1

2

Total

13

7(a) H_0 : No association between the performances at KS3 and GCE

O_i	E_i	$O_i - E_i$	X^2
60	63.55	-3.55	0.1983
55	44.64	10.36	2.4043
40	46.81	-6.81	0.9907
55	51.25	3.75	0.2744
32	36.00	-4.00	0.4444
38	37.75	0.25	0.0017
47	46.33	0.67	0.0097
31	32.54	-1.54	0.0733
35	34.13	0.87	0.0222
43	43.87	-0.87	0.0173
26	30.82	-4.82	0.7527
38	32.31	5.69	1.0005

$$X^2 = 6.1897$$

$$\nu = 3 \times 2 = 6 \Rightarrow \chi_{90\%}^2 = 10.645$$

Do not reject H_0
 No evidence to suggest an association between KS3 results and GCE grades at 10% level of significance.

(b) More of the students achieving level 7 at KS3 gain grade A's at GCE than expected.

B1

M1

M1

M1

M1

A1

B1B1✓

E1✓

E1

Total

E_i

$O_i - E_i$

$(O_i - E_i)^2 / E_i$

Σ

AWFW 6.05 – 6.35

on their ν

9

1

10

<p>1 H_0: condition independent of treatment H_1: condition dependent upon treatment</p> <p>Totals: 66, 84, 75, 75</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>O</th> <th>E</th> <th>$O - E - 0.5$</th> <th>$\frac{(O - E - 0.5)^2}{E}$</th> </tr> </thead> <tbody> <tr> <td>20</td> <td>33</td> <td>12.5</td> <td>4.7348</td> </tr> <tr> <td>55</td> <td>42</td> <td></td> <td>3.7202</td> </tr> <tr> <td>46</td> <td>33</td> <td></td> <td>4.7348</td> </tr> <tr> <td>29</td> <td>42</td> <td></td> <td>3.7202</td> </tr> </tbody> </table> <p>$\chi^2 = 16.91$</p> <p>$\chi^2_{5\%}(1) = 3.841 < 16.91$ Reject H_0</p> <p>Evidence to suggest that the condition of the patients may be dependent upon the treatment that they received</p>	O	E	$ O - E - 0.5$	$\frac{(O - E - 0.5)^2}{E}$	20	33	12.5	4.7348	55	42		3.7202	46	33		4.7348	29	42		3.7202	<p>B1</p> <p>B1</p> <p>M1A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>B1✓</p> <p>A1✓</p> <p>E1✓</p>	<p>for E_i attempted, correctly</p> <p>for use of Yates' correction</p> <p>final column</p> <p>allow 16.9 If no Yates' correction: possible M1A1M0M1A0 If 0.5 incorrectly used: possible M1A1M1M1A0</p> <p>for χ^2 on their ν iff H_0 stated correctly dependent on third M1</p>	<p>10</p>
	O	E	$ O - E - 0.5$	$\frac{(O - E - 0.5)^2}{E}$																			
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